Table of Contents

How to Use This Addenda

Make sure you're ready to teach by noting the **Necessary Materials and Pre-Lesson Prep** you will need to gather or complete prior to the lesson

Find high-leverage instructional moves in the **Lesson Look Fors**. This is what leaders should see when observing your instruction

Note how your lesson objective ties to your state **Standards**

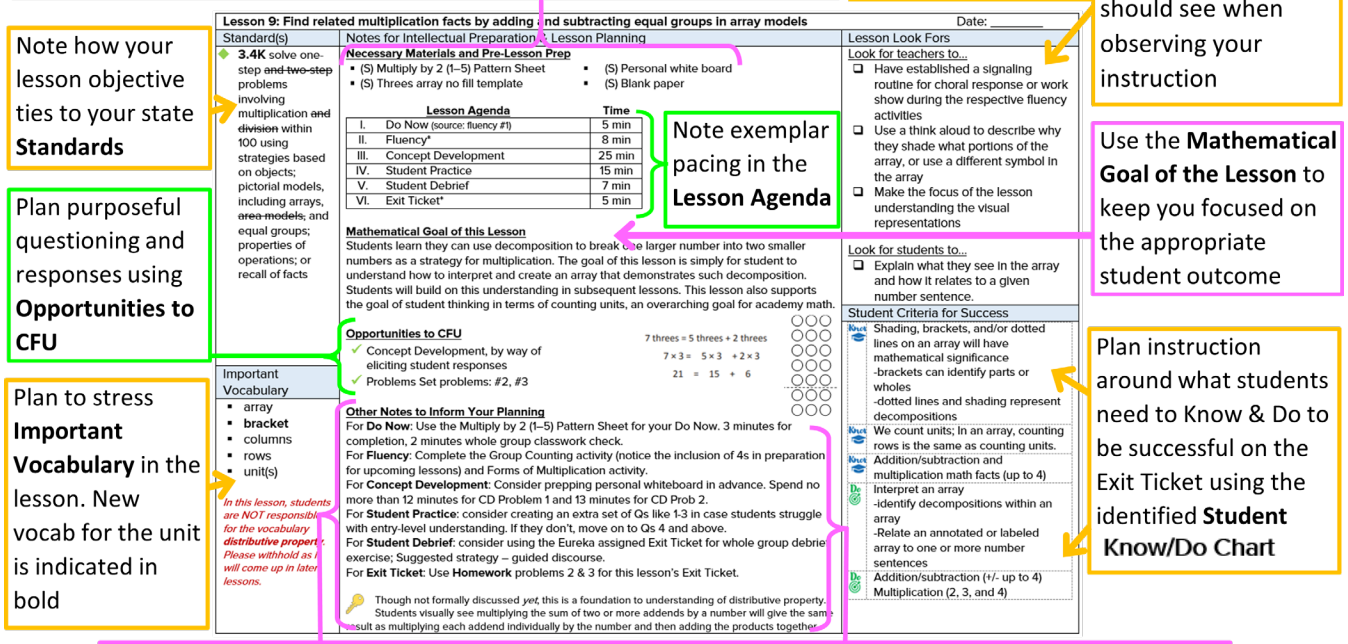
Note exemplar pacing in the **Lesson Agenda**

Use the **Mathematical Goal of the Lesson** to keep you focused on the appropriate student outcome

Plan purposeful questioning and responses using **Opportunities to CFU**

Plan to stress **Important Vocabulary** in the lesson. New vocab for the unit is indicated in bold

Plan instruction around what students need to Know & Do to be successful on the Exit Ticket using the identified **Student Know/Do Chart**



Lesson 9: Find related multiplication facts by adding and subtracting equal groups in array models Date: _____

Standard(s)

3.4K solve one-step ~~and two-step~~ problems involving multiplication ~~and division~~ within 100 using strategies based on objects; pictorial models, including arrays, ~~area models,~~ and equal groups; properties of operations; or recall of facts

Important Vocabulary

- array
- **bracket**
- columns
- rows
- unit(s)

*In this lesson, students are NOT responsible for the vocabulary **distributive property**. Please withhold as it will come up in later lessons.*

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- (S) Multiply by 2 (1–5) Pattern Sheet
- (S) Threes array no fill template
- (S) Personal white board
- (S) Blank paper

Lesson Agenda

		Time
I.	Do Now (source: fluency #1)	5 min
II.	Fluency*	8 min
III.	Concept Development	25 min
IV.	Student Practice	15 min
V.	Student Debrief	7 min
VI.	Exit Ticket*	5 min

Mathematical Goal of this Lesson

Students learn they can use decomposition to break the larger number into two smaller numbers as a strategy for multiplication. The goal of this lesson is simply for student to understand how to interpret and create an array that demonstrates such decomposition. Students will build on this understanding in subsequent lessons. This lesson also supports the goal of student thinking in terms of counting units, an overarching goal for academy math.

Opportunities to CFU

- ✓ Concept Development, by way of eliciting student responses
- ✓ Problems Set problems: #2, #3

7 threes = 5 threes + 2 threes
7 × 3 = 5 × 3 + 2 × 3
21 = 15 + 6

Other Notes to Inform Your Planning

For **Do Now**: Use the Multiply by 2 (1–5) Pattern Sheet for your Do Now. 3 minutes for completion, 2 minutes whole group classwork check.
For **Fluency**: Complete the Group Counting activity (notice the inclusion of 4s in preparation for upcoming lessons) and Forms of Multiplication activity.
For **Concept Development**: Consider prepping personal whiteboard in advance. Spend no more than 12 minutes for CD Problem 1 and 13 minutes for CD Prob 2.
For **Student Practice**: consider creating an extra set of Qs like 1-3 in case students struggle with entry-level understanding. If they don't, move on to Qs 4 and above.
For **Student Debrief**: consider using the Eureka assigned Exit Ticket for whole group debrief exercise; Suggested strategy – guided discourse.
For **Exit Ticket**: Use **Homework** problems 2 & 3 for this lesson's Exit Ticket.

Though not formally discussed *yet*, this is a foundation to understanding of distributive property. Students visually see multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the products together.

Lesson Look Fors

Look for teachers to...

- ❑ Have established a signaling routine for choral response or work show during the respective fluency activities
- ❑ Use a think aloud to describe why they shade what portions of the array, or use a different symbol in the array
- ❑ Make the focus of the lesson understanding the visual representations

Look for students to...

- ❑ Explain what they see in the array and how it relates to a given number sentence.

Student Criteria for Success

Know	Shading, brackets, and/or dotted lines on an array will have mathematical significance -brackets can identify parts or wholes -dotted lines and shading represent decompositions
Know	We count units; In an array, counting rows is the same as counting units.
Know	Addition/subtraction and multiplication math facts (up to 4)
Do	Interpret an array -identify decompositions within an array -Relate an annotated or labeled array to one or more number sentences
Do	Addition/subtraction (+/- up to 4) Multiplication (2, 3, and 4)

Find recommended lesson modifications, content knowledge boosters, and/or high-leverage instructional moves that may not be in your Teacher Edition located in **Other Notes to Inform Your Planning**

UNIT SYNOPSIS

Students describe and analyze the inverse relationship between the cubic and cube root functions, graph and write these inverse functions using function notation such as $f^{-1}(x)$, and use composition of these functions to determine if the functions are inverses of one another. Students graph the functions $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ and analyze key attributes such as domain, range, intercepts, symmetries, and maximum and minimum given an interval. Students analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d. Students investigate parameter changes and key attributes in terms of real-world problem situations. Students solve equations involving rational exponents that have real solutions, focusing on cubic and cube root equations. Students formulate and solve equations involving cubic and cube root equations for real-world situations and justify the solutions in terms of the problem situations.

Solving Equations & Inequalities: Solving an equation is the process of rewriting the equation to make what it says about its variable(s) as simple as possible.

- Solving a cube root equation may require that you cube each side of the equation. A cube root equation can be solved by isolating the cube root on one side of the equation, and then raising each side to the third power.

Equivalence: A single quantity may be represented by many different expressions. The facts about a quantity may be expressed in many different equations (or inequalities).

- Corresponding to every power, there is a root. For example, just as there are squares (second powers), there are square roots. Just as there are cubes (third powers), there are cube roots, and so on.
- You can write a radical expression in an equivalent form using a fractional (rational) exponent instead of a radical.

Properties: All the facts of arithmetic and algebra follow from certain properties.

- You can multiply, divide, simplify, and combine like radical expressions using the properties of real numbers.

Functions: A function is a relationship between variables in which each value of the input variable is associated with a unique value of the output variable. Functions can be represented in a variety of ways, such as graphs, tables, equations, or words. Each representation is particularly useful in certain situations. Some important families of functions are developed through transformations of the simplest form of the function.

- The cubic parent function $f(x) = x^3$ can be transformed into a new cubic function by translating, reflecting, stretching, and/or compressing horizontally, vertically, or both.
- The cube root function $f(x) = \sqrt[3]{x}$ is the inverse of the cubic function $g(x)$ unlike $h(x) = \sqrt{x}$, the domain and range of $f(x) = \sqrt[3]{x}$ are all real numbers.

Misconceptions:

- Some students may think that the cube root of a negative number is imaginary rather than understanding that the cube root of a negative number is just the negative number.

Underdeveloped Concepts:

- Some students may struggle with combining radicals. Students learn briefly in Algebra 1 that like radicals can be combined if they are multiplied but not different radicals (ex: $\sqrt[3]{2} \times \sqrt[3]{3}$). With rational exponents, like bases can be combined with different radicals.

Key Questions:

- What relationships exist between a function and its inverse?
- What graphs, key attributes, and characteristics are unique to cubic functions? Or cube root functions?
- Cubic functions are characterized by constant third differences (the rate of change of the rate of change of the function changes at a constant rate) and can be used to describe, model, and make predictions about situations.
- Cubic root functions are characterized as inverses of cubic functions (under appropriate domain restrictions) and can be used to describe, model, and make predictions about situations.
- What are the key attributes to the parent cubic function? Or the parent cube root function?

CONTENT STANDARDS

Below are the standards addressed in this unit.

Readiness Standards	Supporting Standards
2A.2A Graph the functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$,~~ $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, ~~$f(x) = b^x$, $f(x) = \lvert x \rvert$,~~ and ~~$f(x) = \log_b(x)$ where b is 2, 10, and e,~~ and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.	**2A.2B** Graph and write the inverse of a function using notation such as $f^{-1}(x)$.
2A.7H Solve equations (and inequalities) involving rational exponents.	**2A.6A** Analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.
	2A.6B Solve cube root equations (and inequalities) that have real roots.
	2A.7G Rewrite radical expressions that contain variables to equivalent forms.

NOTE: AP readiness may necessitate extensions from TEKS.

Focus on Disciplinary Literacy 	Mathematical Process Standard (F) – Analyze mathematical relationships to connect and communicate mathematical ideas.
	Mathematical Process Standard (G) – Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Lessons → / Math Supports	L1	L2	L3	L4	L5	L6	L7	L8	L9	L10
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections	✓		✓				✓			✓
makes use of graphic organizers	Graphic Organizers				✓	✓	✓	✓		✓	
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives										
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids			✓			✓	✓	✓	✓	
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports	✓	✓	✓		✓	✓		✓	✓	
includes strategies that support language development											
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share			✓				✓			
includes sentence stems to support students with explanations	- Sentence Stems	✓	✓	✓		✓	✓				
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics										
includes websites or equipment that enhances the lesson	Technological Support	✓		✓		✓	✓	✓	✓	✓	
content can be presented in different forms											
uses hands-on tools or manipulatives to represent the math	- Concrete										
uses drawings to represent the math	- Pictorial	✓	✓	✓		✓	✓	✓	✓	✓	
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

The EFFL Model

Before You EFFL!

Here are helpful resources that you guide you in the right direction before your EFFL lesson!

Why Should We EFFL?

The article advocates for the Experience First, Formalize Later (EFFL) teaching model, emphasizing its effectiveness in fostering deep understanding and flexible thinking in students. The author compares traditional teaching to a game of "Simon Says," where students merely mimic instructions without grasping underlying concepts. In contrast, EFFL encourages students to engage actively with problems, enhancing their ability to understand and apply math concepts creatively.

Tips for Lesson Planning

The article offers practical advice for effective lesson planning beyond the exhaustive and overly detailed approaches often emphasized during teacher training. It underscores the importance of thoughtful preparation but rejects the notion that teachers need to script every minute or detail of a class session.

Making the Most of Your EFFL Lesson Debrief

The article discusses the significance of the debriefing phase in the Experience First, Formalize Later (EFFL) lesson model, emphasizing its role in reinforcing learning and highlighting student contributions. The debrief session is seen as crucial for integrating academic vocabulary, emphasizing key lesson understandings, and valuing students' mathematical insights.

While You EFFL!

While each lesson may be unique in context and skills, all lessons benefit from the following practices:

Teacher Look Fors:

- ❑ Utilizing the Do Now to spark students' interest in the Activity.
- ❑ Use questioning to promote small group discussion and exploration, guided by monitoring questions.
- ❑ Connects Experience First to formal concepts using a colored pencil/pen to take notes along the margin during the Debrief.
- ❑ Facilitates whole-class discussions for students to reflect, share insights, and provides feedback that reinforces key concepts.
- ❑ Tracks time to adapt lesson pacing and support based on student response and engagement.

Students Look Fors:

- ❑ In the Activity, students engage in group work and discourse.
- ❑ Exploring the activity, testing hypotheses and approaches (trial & error).
- ❑ Take notes on key ideas and concepts using different colored pencil/pen to take notes along the margin.
- ❑ Share thoughts and ideas that demonstrate their approach to their work.

Other considerations

- During the **Experience First** phase, if most of your students seem stuck or disengaged, take a moment to pause, reset, and provide clear instructions. Some problems of the Activity are more suitable to do a whole-class discussion as a means to save some instructional time for Student Practice or the Exit Ticket. You are encouraged to adapt the EFFL (Experience First, Formalize Later) process to meet your students' needs while maintaining a focus on student-centered instruction.

ROADMAP

AT A GLANCE: Unit 8 – Cubic and Radical Functions and Equations			
Day	Date	Lesson	Lesson Title
1		1	Roots and Radical Expressions
2		2	Multiplying and Dividing Radical Expressions
3		3	Adding and Subtracting Radical Expressions
4		4	Operations with Rational Exponents
5			*Unit 8 Success Day 1 – Review & Reteach topics from 8.1 – 8.4*
6		5	Attributes and Transformations of Cubic Functions
7		6	Attributes and Transformations of Cube Root Functions
8		7	Modeling with Cubic and Cube Root Functions
9		8	Cubic and Cube Root Functions as Inverses
10		9	Solving Cube Root Equations and Inequalities
11			*Unit 8 Success Day 2 – Review & Reteach topics from 8.5 – 8.9*
12		10	Solving Equations and Inequalities Involving Rational Exponents
13			*Unit 8 Success Day 3 – Unit Assessment Review*
14			End of Unit 8 Assessment

Date: _____

Lesson 1: Roots and Radical Expressions

Standard(s)

- **2A.7G** Rewrite radical expressions that contain variables to equivalent forms.

Important Vocabulary

- **nth root**
- **cube root**
- **principal root**
- **radicand**
- **index**
- **radical form**
- **radical expression**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will find real-number roots and simplify radical expressions.

Opportunities to CFU

✓ After the INM

✓ Student Practice #2-7

Other Notes to Inform Your Planning

In this lesson, students will find real-number roots (including square roots, cube roots, and fourth roots) and simplify radical expressions (including square roots and cube roots). Students have previous experience with square roots from Algebra 1 and Geometry.

Focus on Disciplinary Literacy

INM

The value of n effects how many real-roots will have and the value of n will continue to influence much of the material in the unit.

Lesson Look Fors

Look for teachers to…

- ❑ Connect students to their work with squares and square roots in Algebra 1 to higher indexed radicals.
- ❑ Focus students on how the index being even or odd will determine the simplified solutions with radicals.

Look for students to…

- ❑ Find square, cube, and fourth roots of a number.
- ❑ Simplify radical expressions and use radicals to solve contextual problems.

Student Know/Do Chart

Students can

Do Simplify a radical expression for any nth root given.

Know How to define a radical expression and interpret its principal nth root.

Date: _____

Lesson 2: Multiplying and Dividing Radical Expressions

Standard(s)

◆ **2A.7G** Rewrite radical expressions that contain variables to equivalent forms.

Important Vocabulary

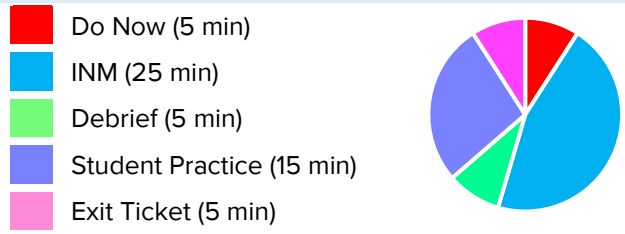
- radicand
- index
- radical expression
- **multiplication property of radicals**
- **division property of radicals**
- simplest radical form
- **rationalize the denominator**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:



Mathematical Goal of this Lesson

In this lesson, students will multiply and divide radical expressions and write the result in simplest form.

Opportunities to CFU

✓ INM #1-5
✓ Debrief
✓ Student Practice #1-5

Other Notes to Inform Your Planning

In this lesson, students will multiply and divide radical expressions and write the results in simplest form. When multiplying and dividing radical expressions, you must first ensure their index is the same. If their index is the same, multiply or divide the coefficients together and multiply or divide the radicands together. The result should then be simplified for the final answer.

When multiplying and dividing radical expressions, a radical may result in the denominator of a fraction. In this case, the denominator must be rationalized. Multiply the fraction to simplify the radical from the denominator and then simplify.

Focus on Disciplinary Literacy

INM & Student Practice

Lesson Look Fors

Look for teachers to…

- ❑ Have students explain what the multiplication and quotient properties of radicals mean in their own terms.
- ❑ Give students real world applications of multiplying and dividing radical expressions.

Look for students to…

- ❑ Simplify radical expressions and rationalize the denominator when required.

Student Know/Do Chart

Students can

Do: Multiply and divide radical expressions and write their expressions in simplest form with rational denominators.

Know: A radical expression in simplest form has no radicands can be factored and reduced, no radicands contain decimals, and no radicals appear in the denominator of a fraction.

Date: _____

Lesson 3: Adding and Subtracting Radical Expressions

Standard(s)

◆ **2A.7G** Rewrite radical expressions that contain variables to equivalent forms.

Important Vocabulary

- **addition property of radicals**
- **like radicals**
- simplest radical form
- rational
- irrational

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will add and subtract radical expressions and write the result in simplest form.

Opportunities to CFU

✓ INM problem #1-4
✓ Debrief
✓ Student Practice #1-6

Other Notes to Inform Your Planning

Focus on Disciplinary Literacy

INM & Student Practice

In this lesson, students will add and subtract radical expressions and write the result in simplest form. When adding and subtracting radical expressions, you must first ensure their index and radicand is the same. If they are, add or subtract their coefficients and keep the radicand the same. The result should then be simplified for the final answer.

When multiplying and dividing radical expressions, there may be radicals that are not in simplest form. Simplify each radical. There may be "like radicals" that could not have been seen without simplifying.

Lesson Look Fors

Look for teachers to…

- ❑ Connect adding and subtracting radicals to combining like terms from polynomial expressions.
- ❑ Focus students on factoring to simplify the radicals and combine as many as possible.

Look for students to…

- ❑ Reduce radical expressions so they can be combined with other radicals by adding or subtracting.
- ❑ Simplify a variety of radical expressions and know when some can't be simplified further.

Student Know/Do Chart

Students can

Do Simplify radical expressions using addition and subtraction.

Know Radicals can only be added or subtracted if they have the same index and the same radicand.

Date: _____

Lesson 4: Operations with Rational Exponents

Standard(s)

◆ **2A.7G** Rewrite radical expressions that contain variables to equivalent forms.

Important Vocabulary

- radicand
- index
- radical expression
- **rational exponent**
- **radical form**
- **Exponential form**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will simplify and perform operations on expressions with rational exponents.

Opportunities to CFU

✓ INM #1-4
✓ Debrief
✓ Student Practice #1-5

Other Notes to Inform Your Planning

In this lesson, students will simplify and perform operations on expressions with rational exponents. The beginning of the lessons explores how the nth root of a number relates to the same number raised to the $\frac{1}{n}$ power. Students will then reinforce the relationship $\sqrt[n]{a} = a^{\frac{1}{n}}$ by proving algebraically $\sqrt{a} = a^{\frac{1}{2}}$ and $\sqrt[3]{a} = a^{\frac{1}{3}}$. Additionally, students will be shown $\sqrt[n]{a^m} = (\sqrt[n]{a})^m = a^{\frac{m}{n}}$. Radical expressions can be converted to exponential expressions by using rational exponents. One advantage to using rational exponents is that you can simplify radical expressions by converting between the two forms and applying exponent and radical properties where applicable.

Focus on Disciplinary Literacy

Do Now & INM

Lesson Look Fors

Look for teachers to…

❑ Be sure students can convert between radical and exponential forms fluently to then be able to simplify expressions using exponent rules.
❑ Students should use what they know about exponential properties to simplify radical expressions in exponential form.

Look for students to…

❑ Convert radical expressions into expressions with rational exponents and then combine or simplify using exponent properties.

Student Know/Do Chart

Students can

Do Perform operations with rational exponents to simplify radical expressions in a new way.

Know Radical expressions can be rewritten using rational exponents as with $\sqrt[n]{a^m} = (\sqrt[n]{a})^m = a^{\frac{m}{n}}$.

Date: _____

Lesson 5: Attributes and Transformations of Cubic Functions

Standard(s)

- **2A.2A** Graph the functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$,~~ $f(x) = x^3$, ~~$f(x) = \sqrt[3]{x}$, $f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e,~~ and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.
- **2A.6A** Analyze the effect on the graphs of $f(x) = x^3$ ~~and $f(x) = \sqrt[3]{x}$~~ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.

Important Vocabulary

- table of values
- **cubic parent function**
- **symmetry**
- reflection in the x-axis
- reflection in the y-axis
- transformation

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (20 min)
- Debrief (10 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will graph and analyze the key attributes of cubic functions and their transformations.

Opportunities to CFU

- ✓ INM problems #1
- ✓ Debrief
- ✓ Student Practice #1-5

Other Notes to Inform Your Planning

In this lesson, students will begin by graphing and analyzing the parent cubic function $f(x) = x^3$. The transformed cubic function is $g(x) = a(b(x - h)^3) + k$ where a is vertical reflections, stretches and compressions, b is horizontal reflections, stretches and compressions, h is horizontal translations, and k is vertical translations. This lesson is best suited for groups of 3-4 students to encourage mathematical discourse.

Focus on Disciplinary Literacy

INM & Student Practice

Lesson Look Fors

Look for teachers to…

- ❑ Remind students of their work in the prior unit with polynomial functions and their graphs.
- ❑ Note for students that the classic "N" shape of cubic curves is only present when there are a variety of transformations happening at once.

Look for students to…

- ❑ Complete tables of coordinates and sketch curves of cubic functions with specific transformations.
- ❑ Connect all of the key attributes of cubic functions and their graphs in the debrief using the graphic organizer in the INM.

Student Know/Do Chart

Students can

Do: Describe the transformation of g based on $a, b, h,$ and k on the parent function $f(x) = x^3$ when $g(x) = a(b(x - h)^3) + k$.

Know: Transformations of a cubic function can involve horizontal or vertical translation, reflections across the x- or y-axis, and vertical or horizontal stretch or compression.

Date: _____

Lesson 6: Attributes and Transformations of Cube Root Functions

Standard(s)

- **2A.2A** Graph the functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$, $f(x) = x^3$~~, $f(x) = \sqrt[3]{x}$, ~~$f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e,~~ and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.
- **2A.6A** Analyze the effect on the graphs of ~~$f(x) = x^3$~~ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.

Important Vocabulary

- table of values
- **cube root parent function**
- symmetry
- reflection in the x-axis
- reflection in the y-axis
- transformation

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (15 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will graph and analyze the key attributes of cube root functions and their transformations.

Opportunities to CFU

- ✓ INM #1-5
- ✓ Debrief
- ✓ Student Practice #1-5

Other Notes to Inform Your Planning

In this lesson, students will begin by graphing and analyzing the parent cube root function $f(x) = \sqrt[3]{x}$. The transformed cube root function is $g(x) = a\sqrt[3]{b(x-h)} + k$ where a is vertical reflections, stretches and compressions, b is horizontal reflections, stretches and compressions, h is horizontal translations, and k is vertical translations.

The key features and attributes of a cube root functions, and its transformations are similar to cubic functions. The relation between the two can be used to form new connections during this lesson.

Focus on Disciplinary Literacy

INM & Student Practice

Lesson Look Fors

Look for teachers to…

- ❑ Remind students of their work in the prior lesson with cubic functions and their graphs.
- ❑ Note for students that cube root is a function so the curves they sketch should pass the vertical line test.

Look for students to…

- ❑ Complete tables of coordinates and sketch curves of cube root functions with specific transformations.
- ❑ Connect all of the key attributes of cube root functions and their graphs in the debrief using the graphic organizer in the INM.

Student Know/Do Chart

Students can

Do: Describe the transformation of g based on $a, b, h,$ and k on the parent function $f(x) = \sqrt[3]{x}$ when $g(x) = a\sqrt[3]{b(x-h)} + k$.

Know: Transformations of a cube root function can involve horizontal or vertical translation, reflections across the x- or y-axis, and vertical or horizontal stretch or compression.

Date: _____

Lesson 7: Modeling with Cubic and Cube Root Functions

Standard(s)

- **2A.2A** Graph the functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$,~~ $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, ~~$f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e,~~ and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.
- **2A.6A** Analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.

Important Vocabulary

- modeling
- cubic parent function
- cube root function
- transformations
- domain
- range

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will model and interpret real-world situations using cubic and cube root functions and their transformations.

Opportunities to CFU

- ✓ INM problem #1-2
- ✓ Debrief
- ✓ Student Practice #1-3

Other Notes to Inform Your Planning

Focus on Disciplinary Literacy

Do Now, INM & Student Practice

In this lesson, cubic and cube root functions will model real-world scenarios. Students will interpret the data in context of the scenario. These applications of cubic and cube root functions mean the domain and range of the functions will need to be restricted. The domain and range will need to be reasonable in context of the scenario.

Students will need to justify and explain the meaning of their answers in context of the scenario.

Lesson Look Fors

Look for teachers to…

- ❑ Focus students on the context of the problems and have them consider if there needs to be a restriction to the domain or the range to use a cubic or cube root model.

Look for students to…

- ❑ Use tables of values and sketches of the curves to make determinations about the question in each problem.
- ❑ Interpret how transformations caused by changes in information would affect a given model.

Student Know/Do Chart

Students can

Do: Model and interpret real-world situations using cubic and cube root functions and their transformations.

Know: Know some scenarios can be modeled by cubic or cube root functions but the solutions to the function should still be considered for reasonableness in the context of the problem.

Date: _____

Lesson 8: Cubic and Cube Root Functions as Inverses

Standard(s)

- **2A.2B** Graph and write the inverse of a function using notation such as $f^{-1}(x)$.

Important Vocabulary

- cubic parent function
- cube root function
- **inverse relation/function**
- **one-to-one**
- **horizontal line test**
- **vertical line test**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will graph and write the inverse of cubic and cube root functions.

Opportunities to CFU

- ✓ INM problems #1-8
- ✓ Debrief
- ✓ Student Practice #1-6

Other Notes to Inform Your Planning

In this lesson, students will graph and write the inverse of cubic and cube root functions. Students will review inverse functions in the beginning of the lesson. After, students will learn cube root functions and cubic functions are inverses of each other. You can find the inverse of each type of function by interchanging x and y and then solve for y. Students will also verify two functions are the inverse of each other by a composition of functions: $f(g(x)) = x$ and $g(f(x)) = x$.

Focus on Disciplinary Literacy

INM & Student Practice

Lesson Look Fors

Look for teachers to…

- ❑ Support students as they find the inverse by switching the x and y and then resolving for y. Students should use the inverse operation (cube or cube root) to isolate the y.

Look for students to…

- ❑ Verify that cube root and cubic functions are inverses with composition and that both are functions with the vertical and horizontal line tests.
- ❑ Find the inverse of a variety of cubic or cube root functions.

Student Know/Do Chart

Students can

- Do: Test if two functions are inverse of each other or not.
- Do: Find the inverse of a given cubic or cube root function and write it in standard form.
- Know: The inverse of a cubic function is a cube root function and vice versa. Cubic and cube root functions are one-to-one.

Date: _____

Lesson 9: Solving Cube Root Equations and Inequalities

Standard(s)

- **2A.6B** Solve cube root equations (and inequalities) that have real roots.

Important Vocabulary

- **cube root equation**
- **cube root inequality**
- solution
- evaluate

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson students will solve cube root equations and inequalities.

Opportunities to CFU

✓ INM #1-3 ✓ Student Practice #8

Other Notes to Inform Your Planning

In this lesson, students will solve cube root equations and inequalities algebraically and graphically. Students will review solving square root equations in the beginning of the lesson which involves identifying extraneous solutions. Students will solve cube root equations by raising both sides of an equation to the third power.

Focus on Disciplinary Literacy

INM

After, students explore why you do not need to identify if there are extraneous solutions when you raise both sides of an equation to the third power (every number – positive or negative, has a unique cube root).

Lesson Look Fors

Look for teachers to…

- ❑ Pay close attention to students' use of interval and inequality notation as well as correct graphs with shading.
- ❑ Support students in understanding that there are no extraneous solutions when solving cube root equations.

Look for students to…

- ❑ Cube both sides to eliminate cube roots and expand with polynomial multiplication as needed.
- ❑ Represent solutions to cube root inequalities with the proper methods that show the solution is an inequality and not a single value.

Student Know/Do Chart

Students can

Do Solve cube root equations and inequalities.

Know Cube root equations have unique solutions and no extraneous solutions.

Date: _____

Lesson 10: Solving Equations and Inequalities Involving Rational Exponents

Standard(s)

- **2A.7H** Solve equations (and inequalities) involving rational exponents.

Important Vocabulary

- nth root
- radical expression
- index
- radicand
- rational exponent
- **radical form**
- **exponential form**
- **radical equation**
- solution
- evaluate
- extraneous solution

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep

- SE workbook
- graphing calculator

Lesson Structure:

- Do Now (5 min)
- INM (25 min)
- Debrief (5 min)
- Student Practice (20 min)
- Exit Ticket (5 min)

Mathematical Goal of this Lesson

In this lesson, students will solve equations and inequalities involving rational exponents.

Opportunities to CFU

✓ INM #1-6

✓ Student Practice #1-11

Other Notes to Inform Your Planning

In this lesson, students will solve equations involving rational exponents. Each practice question reveals a different challenge to solve the equation. One method of solving equations involving rational exponents is to isolate the expression with the rational exponent and then raise both sides of the equation to the reciprocal of the rational exponent. If you cannot isolate the expression with the rational exponent, you can solve by factoring, raising both sides of the equation to a power that eliminates the fractional exponents, or other problem-solving techniques. While you do not need to verify your solutions with every equation that involves rational exponents, it is important to still do so. You can verify your solutions by graphing both sides of the equation and find the points of intersection.

Lesson Look Fors

Look for teachers to…

- ❑ Focus students on the process of isolating the variable and solving by raising both sides to the reciprocal power.
- ❑ Remind students of skills like factoring to solve and then setting up multiple equations using the zero product property.

Look for students to…

- ❑ Use a combination of skills to isolate the variable in a variety of rational exponent equations and inequalities.
- ❑ If there is an exponent with an even denominator, check for extraneous solutions.

Student Know/Do Chart

Students can

Do: Solve equations and inequalities involving rational exponents.

Know: Exponents can not be distributed to all terms; they must be applied to the entire side of the equation or inequality.

Recommended Unit 8 Success Days Material and Resources

Date: _____

To review topics taught up to 8.4, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

To review roots and radical expressions:

Content Video Lessons:

- Simplifying Cube Root Expressions
- Simplifying Cube Root Expressions (Two Variables)

To review operations of radical expressions:

Content Video Lessons:

- Multiplying Radical Expressions
- Multiplying and Simplifying Cube Root Expressions
- Divide and Simplify Radical Expressions
- Rationalizing the Denominator
- Simplifying Radical Expressions
- Adding and Simplifying Radical Expressions
- Subtracting and Simplifying Radical Expressions
- Adding and Subtracting Radical Expressions
- Converting Rational Exponents and Radicals
- Introduction to Rational Exponents
- Rational Expressions

Desmos Interactive Applets:

- Fractional Exponents

To review topics taught up to 8.9, use the following resources. Your exit ticket data should be used to determine individualized needs. The resources can be used in small groups, whole groups, or independent groups and be integrated with other classroom routines, like computer aligned practice and teacher-led groups.

To review cubic and cube root functions:

Desmos Interactive Applets:

- Transformations of Cubic Functions
- Transformations Cube Root Functions

Content Video Lessons:

- Attributes and Transformations of Cubic Functions
- Graphing Cubic Function Transformations (Desmos)
- Graph and Describe Transformation of Cubic Function
- Attributes and Transformations of Cube Root Functions
- Graphing Cube Root Functions
- Finding Inverse of a Cube Root Function
- Verifying Inverse Functions by Composition

To review solving cubic and cube root equations and inequalities:

Content Video Lessons:

- Solving Cube Root Equations
- Solving Equations with Cube Roots
- Cube Root Inequalities
- Solving an Equation with Rational Exponent
- Solving Radical Equations and Inequalities

Date: _____

Unit 8 Exam

Standard(s)	Notes for Intellectual Preparation & Lesson Planning
◆ **2A.2A** Graph the functions ~~$f(x) = \sqrt{x}$~~, ~~$f(x) = 1/x$~~, $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, ~~$f(x) = b^x$~~, ~~$f(x) = \lvert x \rvert$~~, and ~~$f(x) = \log_b(x)$~~ ~~where b is 2, 10, and e~~, and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval. **Problem #6** ◆ **2A.7H** Solve equations (and inequalities) involving rational exponents. **Problems #10ac** ◆ **2A.2B** Graph and write the inverse of a function using notation such as $f^{-1}(x)$. **Problem #1** ◆ **2A.6A** Analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d. **Problems #5, 7** ◆ **2A.6B** Solve cube root equations (and inequalities) that have real roots. **Problem #10b** ◆ **2A.7G** Rewrite radical expressions that contain variables to equivalent forms. **Problems #2, 3, 4, 8, 9**	**Necessary Materials and Pre-Lesson Prep** ▪ Algebra 2 Unit 8 Exam ▪ Assessment Companion for Algebra 2 Unit 8 Exam found on Curriculum Corner **Notes to Inform Your Planning** Review the Unit 8 Exam on Curriculum Corner. Internalize & create an exemplar for the assessment prior to teaching the unit as part of unpacking the unit. Use the exemplar to spar with the answer key provided on Curriculum Corner. Administer the Unit 8 Exam following guidance on the Scope & Sequence to ensure the scoring deadline is met.

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**
2A.2A Graph the functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$,~~ $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, ~~$f(x) = b^x$, $f(x) = |x|$, and $f(x) = \log_b(x)$ where b is 2, 10, and e,~~ and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval.	Including, but not limited to: • Representations of functions, including graphs, tables, and algebraic generalizations o Cubic, $f(x) = x^3$ o Cube root, $f(x) = \sqrt[3]{x}$ • Connections between representations of families of functions • Comparison of similarities and differences of families of functions	• Grade Level(s): o The notation $\Re$ represents the set of real numbers, and the notation Z represents the set of integers. o Algebra I studied parent functions f(x) = x, $f(x) = x^2$, and $f(x) = b^x$ and their key attributes. o Precalculus will study polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions. o Various mathematical process standards will be applied to this student expectation as appropriate.
2A.7H Solve equations (and inequalities) involving rational exponents.	Including, but not limited to: • Laws (properties) of exponents o Product of powers (multiplication when bases are the same): $a^m \cdot a^n = a^{m+n}$ o Quotient of powers(division when bases are the same): $\frac{a^m}{a^n} = a^{m-n}$ o Power to a power: $(a^m)^n = a^{mn}$ o Negative exponent: $a^{-n} = \frac{1}{a^n}$ o Rational exponent: $a^{\frac{1}{n}} = \sqrt[n]{a}$, $a^{\frac{m}{n}} = \sqrt[n]{a^m}$ o Equations when bases are the same: $a^m = a^n \rightarrow m = n$ • Solving equations with rational exponents o Isolation of base and power using properties of algebra o Exponentiation of both sides by reciprocal of power of base o Simplification to obtain solution o Verification of solution • Real-world problem situations modeled by equations involving rational exponents • Justification of reasonableness of solutions in terms of real-world problem situations	• Grade Level(s): o Prior grade levels simplified numeric expressions, including integral and rational exponents. o Algebra II introduces equations involving rational exponents. o Various mathematical process standards will be applied to this student expectation as appropriate.
2A.2B Graph and write the inverse of a function using notation such as $f^{-1}(x)$.	Including, but not limited to: • Inverse of a function – function that undoes the original function. When composed $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$. • Inverses of functions on graphs o Symmetric to $y = x$ • Inverses of functions in tables o Interchange independent (x) and dependent (y) coordinates in ordered pairs	• Grade Level(s): o Algebra II introduces inverse of a function. o Various mathematical process standards will be applied to this student expectation as appropriate.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**

	• Inverses of functions in equation notation
	  o Interchange independent (x) and dependent (y) variables in the equation, then solve for y
	• Inverses of functions in function notation
	  o $f^{-1}(x)$ represents the inverse of the function $f(x)$.	

2A.6A Analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d.	Including, but not limited to:
	• General form of the cubic and cube root functions
	  o Cubic
	  o Cube root
	• Representations with and without technology
	  o Graphs
	  o Tables
	  o Verbal descriptions
	  o Algebraic generalizations
	• Effects on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when parameters a, b, c, and d are changed in $f(x) = a(b(x - c))^3 + d$ and $f(x) = a\sqrt[3]{b(x - c)} + d$
	  o Effects on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$, when $f(x)$ is replaced by $af(x)$ with and without technology
	  o Effects on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$, when $f(x)$is replaced by $f(bx)$ with and without technology
	  o Effects on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$, when $f(x)$ is replaced by $f(x - c)$ with and without technology
	  o Effects on the graphs of $f(x) = x^3$, and $f(x) = \sqrt[3]{x}$, when $f(x)$ is replaced by $f(x) + d$ with and without technology
	Connections between the critical attributes of transformed function and $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$
	  o Determination of parameter changes given a graphical or algebraic representation
	  o Determination of a graphical representation given the algebraic representation or parameter changes
	  o Determination of an algebraic representation given the graphical representation or parameter changes
	Descriptions of the effects on the domain and range by the parameter changes
	Effects of multiple parameter changes
	  o Mathematical problem situation & Real-world problem situation
		• Grade Level(s):
		  o Algebra I determined effects on the graphs of the parent functions, $f(x) = x$ and $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d.
		  o Algebra II introduces the cubic and cube root functions and their transformations.
		  o Various mathematical process standards will be applied to this student expectation as appropriate.

2A.6B Solve cube root equations (and inequalities) that have real roots.	Including, but not limited to:
	• Application of laws (properties) of exponents
	• Application of cube roots to solve cubic equations
	• Applications of cubics to solve cube root equations
	• Reasonableness of solutions
	  o Substitution of solutions into original problem
	  o Graphical analysis
	• Mathematical problem situations
	• Real-world problem situations
		• Grade Level(s):
		  o Algebra II introduces cubic and cube root functions and solving cube root equations.
		  o Various mathematical process standards will be applied to this student expectation as appropriate.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**
2A.7G Rewrite radical expressions that contain variables to equivalent forms.	Including, but not limited to: • Laws (properties) of exponents • Radical expression – expression that contains a radical symbol o The symbol $\sqrt[n]{\ }$ is called a radical. o The root number in the bend of the radical symbol is called the index. For $\sqrt[n]{p}$, the index is *n*. o The expression under the radical symbol is called the radicand. For $\sqrt[n]{p}$, the radicand is *p*. o If no index is indicated on the radical symbol, it is understood to be a square root. o Sample radical expressions o The radical symbol represents a fractional exponent, and the expression can be rewritten with the fractional exponent. • Simplification of radical expressions o All coefficients and variables should be written as factors in power form, $56xy = 2^3 \cdot 7^1x^3y^5$ o The root is taken by removing groups from the radicand according to the index value. (Hint: Divide the index into the power to determine power on the number or variable taken out of the radicand. Any remainder will be left in the radicand.) o The root may also be taken by writing the expression with a fractional exponent and simplifying using the power rule of exponents, $(ab)^n = a^nb^n$. o If the index of a radical expression is even and the power of the variable simplified out of the radical and is odd, then the variable, including its power, must be represented using absolute value. o A factor of negative one in radicands with even indices, 2, 4, 6, …, have no real solutions. o A factor of negative one in radicands with odd indices, 1, 3, 5, …, have a factor of negative one in the simplified answer.	

VERTICAL STANDARDS

This section details the **progression** of key student expectations (SEs) in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

Algebra 1	Algebra 2	Pre-Calculus
• **A.7A** Graph quadratic functions on the coordinate plane and use the graph to identify key attributes, if possible, including x-intercept, y-intercept, zeros, maximum value, minimum values, vertex, and the equation of the axis of symmetry. • **A.7C** Determine the effects on the graph of $f(x) = x^2$ when $f(x)$ is replaced by $af(x)$, $f(x) + d$, $f(bx)$, and $f(x - c)$ for specific values of a, b, c, and d. • **A.11A** Simplify numerical radical expressions involving square roots. • **A.11B** Simplify numeric and algebraic expressions using the laws of exponents, including integral and rational exponents.	• **2A.2A** Graph the functions ~~$f(x) = \sqrt{x}$, $f(x) = 1/x$,~~ $f(x) = x^3$, $f(x) = \sqrt[3]{x}$, ~~$f(x) = b^x$, $f(x) = \lvert x \rvert$, and $f(x) = \log_b(x)$ where b is 2, 10, and e,~~ and, when applicable, analyze the key attributes such as domain, range, intercepts, symmetries, asymptotic behavior, and maximum and minimum given an interval. • **2A.2B** Graph and write the inverse of a function using notation such as $f^{-1}(x)$. • **2A.7H** Solve equations (and inequalities) involving rational exponents. • **2A.6A** Analyze the effect on the graphs of $f(x) = x^3$ and $f(x) = \sqrt[3]{x}$ when $f(x)$ is replaced by $af(x)$, $f(bx)$, $f(x - c)$, and $f(x) + d$ for specific positive and negative real values of a, b, c, and d. • **2A.6B** Solve cube root equations (and inequalities) that have real roots. • **2A.7G** Rewrite radical expressions that contain variables to equivalent forms.	• **(2.G)** Graph functions, including ~~exponential, logarithmic, sine, cosine, rational,~~ polynomial, and power functions and their transformations, including $af(x)$, $f(x) + d$, $f(x - c)$, $f(bx)$ for specific values of a, b, c, and d, in mathematical and real-world problems. • **(2.I)** Determine and analyze the key features of ~~exponential, logarithmic, rational,~~ polynomial, ~~power, trigonometric, inverse trigonometric, and piecewise defined~~ functions, ~~including step functions~~ such as domain, range, symmetry, relative maximum/minimum, zeros, asymptotes, and intervals over which the function is increasing/decreasing. • **(2.J)** Analyze and describe end behavior of functions, including ~~exponential, logarithmic, rational~~, polynomial, and power functions, using infinity notation to communicate this characteristic in mathematical and real-world problems. • **(2.N)** Analyze situations modeled by functions, including ~~exponential, logarithmic, rational,~~ polynomial, and power functions, to solve real-world problems. • **(5.J)** Solve polynomial equations with real coefficients by applying a variety of techniques in mathematical and real-world problems. • **(5.K)** Solve polynomial inequalities with real coefficients by applying a variety of techniques and write the solution set of the polynomial inequality in interval notation in mathematical and real-world problems.